Astrophysical Dynamics — HS 09-10
Exercises Class 2 (due Oct 21)

Exercise 1

Molecular clouds are made by a dense core and a diffuse envelope of lower density. Consider a cloud with a temperature of 30 K in both components. The core density is 250 cm^{-3}, while the envelope density can be obtained assuming pressure equilibrium with the surrounding cold neutral medium (CNM: $n_{\mathrm{CNM}} = 50\ \mathrm{cm}^{-3}$, $T_{\mathrm{CNM}} = 90$ K). The relevant heating processes are:

- **cosmic rays** heating:

$$\Gamma_{\mathrm{CR}} = 2 \cdot 10^{-13} \left(\frac{n_{\mathrm{H}_2}}{10^3\ \mathrm{cm}^{-3}} \right) \mathrm{eV\ cm}^{-3}\ \mathrm{s}^{-1}$$

- **carbon ionization**: free electrons ejected from carbon atoms ($n_{\mathrm{C}}/n_H = 3 \cdot 10^{-4}$) by incident UV photons disperse their kinetic energy through collisions, at a rate

$$\Gamma_{\mathrm{CI}} = 10^{-7} \left(\frac{n_{\mathrm{C}}}{10^3\ \mathrm{cm}^{-3}} \right) \mathrm{eV\ cm}^{-3}\ \mathrm{s}^{-1}$$

- **photoelectric heating**: as in carbon ionization, ultraviolet radiation can extract electrons from interstellar dust grains, provinding heating at the rate

$$\Gamma_{\mathrm{PH}} = 3 \cdot 10^{-11} \left(\frac{n_{\mathrm{H}_2}}{10^3\ \mathrm{cm}^{-3}} \right) \mathrm{eV\ cm}^{-3}\ \mathrm{s}^{-1}$$

- **irradiation of dust grains**: lower energy radiation does not extract electrons but raise the temperature of the dust; if collisions between dust and hydrogen molecules are efficient, the effective heating rate is

$$\Gamma_{\mathrm{d}} = 2 \cdot 10^{-9} \left(\frac{n_{\mathrm{H}_2}}{10^3\ \mathrm{cm}^{-3}} \right) \mathrm{eV\ cm}^{-3}\ \mathrm{s}^{-1}$$

What are the cooling rates necessary for the core and the envelope to be in steady state (i.e. thermal balance) at its temperature of 30 K? Note that ultraviolet photons are absorbed as they travel through the envelope, thus they do not contribute to the heating of the core.

Exercise 2

A source emits blackbody radiation with an effective temperature $T_* = 6500$ K.

a) The distance of the source is 1 pc. Assume that absorption on the line of sight comes mainly from dust grains, whose mass fraction in the ISM is 2%, and that their opacity is $\kappa = 100\ \mathrm{cm}^2\ \mathrm{g}^{-1}$. What density of the ISM (averaged on the line of sight) corresponds to the transition from optically thin medium to optically thick? What is the brightness attenuation for $n_{\mathrm{ISM}} = 0.12\ \mathrm{cm}^{-3}$?

b) Assume that the source is now enveloped in a geometrically thin, optically thick layer ($\tau = 100$), and that outside it the optical depth is zero. If the radiation flux through this layer is $10^7\ \mathrm{erg\ cm}^{-2}\ \mathrm{s}^{-1}$, what is the effective temperature of the radiation emerging from it?