

Astrophysical Dynamics — HS 09-10
Exercises Class 3 (DUE OCTOBER 28th!!!)

1

Starting from the monopole term of the gravitational potential (which is the exact potential for a spherically symmetric system),

$$\Phi(r) = -4\pi G \left[\frac{1}{r} \int_0^r \rho(r') r'^2 dr' + \int_r^\infty \rho(r') r' dr' \right]$$

show that Newton I and II follow, i.e.

a) a body that is inside a homogeneous spherical shell of matter experiences no net gravitational force from that shell;

b) the gravitational force exerted by a spherical shell of matter onto any body outside of it is the same as if all the shell's matter were concentrated into a point at its center.

2

The circular velocity on the $z = 0$ plane of an axisymmetric system is given by

$$V_c(R)_{z=0} = \left(R \frac{d\Phi}{dR} \right)_{z=0}^{1/2}$$

where Φ is again the gravitational potential.

Consider now the Miyamoto-Nagai potential-density pair

$$\Phi_{\text{MN}}(R, z) = - \frac{GM_{\text{tot}}}{\left[R^2 + (a + \sqrt{b^2 + z^2})^2 \right]^{1/2}},$$

$$\rho_{\text{MN}}(R, z) = \left(\frac{b^2 M_{\text{tot}}}{4\pi} \right) \frac{aR^2 + (a + 3\sqrt{b^2 + z^2})(a + \sqrt{b^2 + z^2})^2}{\left[R^2 + (a + \sqrt{b^2 + z^2})^2 \right]^{5/2} (b^2 + z^2)^{3/2}}.$$

Suppose you measured the rotation curve $V_c(R)_{z=0}$ of such a system; what would be the inferred density profile $\rho_{\text{spher}}(R)$ if you assumed that the system was spherically symmetric? Show the behavior of ρ_{MN} and ρ_{spher} as $R \rightarrow 0$.