# PPPI HS09
# Exercise Sheet 1

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**Exercise 1** [*Natural Units*]

We explore the use of "natural" units in the following:

a) Find the relationships between energy (GeV) and time (s), length (cm) and grams (g).

b) Calculate the ratio $a_{\text{Bohr}} : \lambda_{\text{Compton}} : r_0$ for an electron. In cgs units, $a_{\text{Bohr}} = \hbar^2/m_e e^2$, $\lambda_{\text{Compton}} = \hbar/m_e c$, $r_0 = \alpha\hbar/m_e c$ (the classical electron radius) and $\alpha = e^2/\hbar c$.

c) The Boltzmann constant is often set to $k_B = 1$. What is the relationship between temperature (K) and energy (GeV)?

**Exercise 2** [*Photon scattering*]

A photon $\gamma$ ($k^2 = 0$) with the 4-momentum $k^\mu = (E, E, 0, 0)$ scatters off a stationary electron $e$. After the scattering, the momentum of the photon is $k'^\mu = (E', E'\cos\Theta, E'\sin\Theta, 0)$. Show that the final energy of the photon $E'$ is given by

$$E' = \frac{E}{1 + \frac{E}{m_e}(1 - \cos\Theta)}.$$

**Exercise 3** [*$2 \to 2$ scattering*]

For a $2 \to 2$ scattering ($1 + 2 \to 3 + 4$) consider the following in the center of momentum frame.

a) Calculate the energies, $E_i^*$, and the momenta before, $|\vec{p}_i|$, and after, $|\vec{p}_i'|$, the scattering, of each particle, and determine their asymptotic ($s \gg m_i^2$) behaviour.

– please turn over –

b) Using the above results, show that the scattering angle $\Theta^*$ is given by

$$\cos\Theta^* = \frac{s(t-u) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{\sqrt{\lambda(s, m_1^2, m_2^2)}\sqrt{\lambda(s, m_3^2, m_4^2)}},$$

with

$$\lambda(s, m_1^2, m_2^2) = (s - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2.$$

Determine that $s$, $t$ and $u$ are not independent by showing that they fulfill the equation $s + t + u = \sum_i m_i^2$.

c) Find the physically valid values for the scattering angle, and thus determine $t_{\min}$ and $t_{\max}$. Calculate the leading terms of $t_{\min}$ in the asymptotic region $s \gg m_i$ both for the general case of unequal masses ($m_i \neq m_{i'}$) and for $m_2 = m_4$.