QFT I
Exercise Sheet 1

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Exercise 1 [*Dirac current*]

The Dirac equations for the spinor ψ and the adjoint spinor $\bar{\psi}$ are [1]

$$i\gamma^\mu \partial_\mu \psi - \frac{mc}{\hbar}\psi = 0 \qquad , \quad i\partial_\mu \bar{\psi}\gamma^\mu + \frac{mc}{\hbar}\bar{\psi} = 0\ . \tag{1}$$

Show that the Dirac current fulfils a continuity equation $\partial_\mu j^\mu = 0$, being $j^\mu = \bar{\psi}\gamma^\mu\psi$.

Exercise 2 [*Pauli equation*]

Consider the non-relativistic limit of the Dirac equation

$$i\hbar\frac{\partial\varphi}{\partial t} = \left(\frac{1}{2m}(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{\pi}) + e\phi\right)\varphi \tag{2}$$

and show that it corresponds to the non-relativistic Pauli equation

$$i\hbar\frac{\partial\varphi}{\partial t} = \left(\frac{1}{2m}(\vec{p} - e\vec{A})^2 - \frac{e\hbar}{2m}\vec{\sigma}\cdot\vec{B} + e\phi\right)\varphi. \tag{3}$$

In addition, considering a weak homogeneous magnetic field, show that eq. (3) describes the anomalous Zeeman effect with the correct electron's gyromagnetic ratio.

Exercise 3 [*Equation of Motion*]

Starting from the Lagrangian density

$$\mathcal{L} = \bar{\psi}(ic\hbar\gamma^\mu\partial_\mu - mc^2)\psi \tag{4}$$

obtain the equations of motion for the two spinors ψ and $\bar{\psi}$.

[1] In what follows, it is convenient to work using natural units, namely $\hbar = c = 1$