QFT I
Exercise Sheet 10

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Exercise 1 [*Dimensional Regularization*]
The following two integrals appear frequently in dimensional regularization. Show that:

$$\int \frac{\mathrm{d}^d p_E}{(2\pi)^d} \frac{1}{(p_E^2+\Delta)^n} = \frac{1}{(4\pi)^{d/2}} \frac{\Gamma\left(n-\frac{d}{2}\right)}{\Gamma(n)} \Delta^{\frac{d}{2}-n}$$

$$\int \frac{\mathrm{d}^d p_E}{(2\pi)^d} \frac{p_E^2}{(p_E^2+\Delta)^n} = \frac{1}{(4\pi)^{d/2}} \frac{d}{2} \frac{\Gamma\left(n-\frac{d}{2}-1\right)}{\Gamma(n)} \Delta^{1+\frac{d}{2}-n}$$

Exercise 2 [*Feynman Parametrization*]
To bring loop integrals with n propagators into a standard form, one usually uses the so-called Feynman parametrization. Show that:

$$\frac{1}{D_1^{a_1}\dots D_n^{a_n}} = \frac{\Gamma(a_1+a_2+\dots+a_n)}{\Gamma(a_1)\Gamma(a_2)\dots\Gamma(a_n)} \times \int_0^1 \mathrm{d}x_1 \dots \int_0^1 \mathrm{d}x_n \frac{\delta(1-x_1-x_2-\dots-x_n)x_1^{a_1-1}x_2^{a_2-1}\dots x_n^{a_n-1}}{[x_1 D_1+\dots+x_n D_n]^{a_1+\dots+a_n}}$$

by using induction.

Exercise 3 [*Power Counting*]
By means of Power Counting, show that the theory described by the following Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2) - \lambda\frac{\phi^3}{3!}$$

is Super-Renormalizable in 4 dimensions.
Hints

- A Field Theory is called Super-Renormalizable if only a finite number of diagrams diverge superficially (logarithmic divergences only).
- Draw all the possible one loop diagrams with an arbitrary number of external legs and show that the only divergent diagram is the two-point bubble (self energy of the scalar field).
- Repeat the same procedure at n loops, observing that the only divergent diagrams are those containing a self energy insertion as a subdiagram.
- In the end you proved that the only divergent diagram is the one loop self energy. Renormalizing it makes the whole theory finite.