QFT I
Exercise Sheet 11

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Exercise 1 [*Structure of the photon two-point function*]
Without explicitly evaluating the loop integral, show that

$$\frac{q^\mu q^\nu}{q^4}\Pi_{2,L}(q) = 0$$

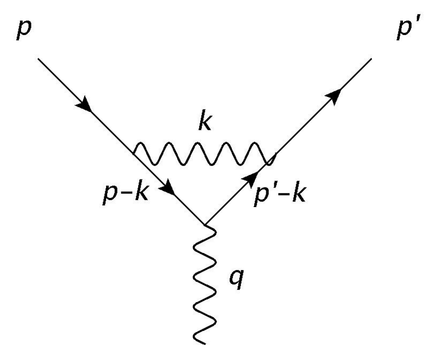
where

$$\Pi_2^{\mu\nu}(q) = (q^2 g^{\mu\nu} - q^\mu q^\nu)\Pi_2(q) + q^\mu q^\nu \Pi_{2,L}(q)$$

and

$$\Pi_2^{\mu\nu}(q) = -e^2\mu^{2\epsilon}\int \frac{\mathrm{d}^d k}{(2\pi)^d}\frac{\mathrm{Tr}\,(\gamma^\mu(\not{k}+m)\gamma^\nu((\not{k}+\not{q})+m))}{(k^2-m^2)((k+q)^2-m^2)}$$

Exercise 2 [*One-loop correction to the QED vertex*]
The aim of this exercise is the computation of the on-shell ($q^2 = 0$, $p^2 = p'^2 = m^2$) one-loop correction to the electromagnetic vertex.



(i) Write the amplitude corresponding to the diagram above, and bring it to the following form

$$i\mathcal{M} = e^3\int \frac{\mathrm{d}^d k}{(2\pi)^d}\bar{u}(p')\frac{\mathcal{Z}}{\mathcal{N}}u(p).$$

(ii) Focusing on the denominator $\mathcal{N}$:

- Use a suitable Feynman parametrization to rewrite the denominator as

$$\frac{1}{\mathcal{N}} = 2\int_0^1 \mathrm{d}x \int_0^1 \mathrm{d}y \int_0^1 \mathrm{d}z \frac{\delta(1-x-y-z)}{D^3},$$

  where

$$D = k^2 - 2k(xp + yp') + i\epsilon.$$

- Complete the square and show that D can be written as

$$D = k'^2 - \Delta + i\epsilon, \qquad k' = k - xp - yp', \qquad \Delta = m^2(x+y)^2.$$

(iii) Show that the numerator $\mathcal{Z}$ can be brought to the form

$$\mathcal{Z} = -2\epsilon^*_\mu(q)\left[\left(-\frac{1}{2}k'^2 + m^2(1-4z+z^2)\right)\gamma^\mu + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\left(2m^2 z(1-z)\right)\right].$$

To do so, use:

- the anticommutation relations for the γ-matrices $\not{p}\gamma^\mu = 2p^\mu - \gamma^\mu \not{p}$,
- the Dirac equation ($\bar{u}(p')\not{p}' = m\bar{u}(p')$, $\not{p}u(p) = mu(p)$),
- the symmetry of the integration over k', which allows the following tensorial replacements in the numerator

$$k'^\mu \to 0, \qquad k'^\mu k'^\nu \to \frac{1}{d}g^{\mu\nu}k'^2,$$

- the symmetry of the integral under the interchange $x \leftrightarrow y$,
- the Gordon identity

$$\bar{u}(p')\gamma^\mu u(p) = \bar{u}(p')\left(\frac{1}{2m}\left(p^\mu + p'^\mu + i\sigma^{\mu\nu}(p'_\mu - p_\mu)\right)\right)u(p).$$

(iv) Using the results obtained in exercise sheet 10, Integrate over the loop momentum k'.

(v) Integrate over x and y

(vi) Write the result in the following way

$$\bar{u}(p')\left(-ie\epsilon^*_\mu(q)\left(F_1(0)\gamma^\mu + \frac{1}{2m}F_2(0)i\sigma^{\mu\nu}q^\nu\right)\right)u(p).$$