# QFT I
# Exercise Sheet 12

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**Exercise 1** [*Form Factor Decomposition*]

Compute the integral

$$I_{\alpha\beta\mu\nu\rho\sigma}(n) := \int \mathrm{d}^D k \frac{k_\alpha k_\beta k_\mu k_\nu k_\rho k_\sigma}{(k^2-\mu^2)^n}$$

using the Passarino-Veltman decomposition into form factors and the symmetry of the integrand under interchange of any two indices. Compute the pole at $D = 4$ for $n = 5$.

**Exercise 2** [*Photon Mass in Two-Dimensional QED*]

Consider massless two-dimensional QED (known as the Schwinger model).

- Calculate the vacuum polarisation at one loop. It is given by

$$-i\Pi_{\mu\nu}(p) = (ie)^2(-i)^2 \int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{\mathrm{Tr}\left((\not{k}-\not{p})\gamma_\nu \not{k}\gamma_\mu\right)}{k^2(k-p)^2}.$$

  We take the dimension of space to be $D = 2-\epsilon$, the trace identities in $2-\epsilon$ dimensions are

$$\mathrm{Tr}(\gamma_\mu\gamma_\nu) = 2g_{\mu\nu}, \qquad \mathrm{Tr}(\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma) = 2(g_{\mu\nu}g_{\rho\sigma} + g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma}).$$

- Determine the full one-loop photon propagator by summing the geometric series of one-particle irreducible one-loop contributions. Make use of the fact that the photon is always coupled to a conserved current. Determine the mass of the photon in this theory from the full one-loop photon propagator.

Figure 1: The one-loop photon propagator is obtained by summing the series below.