QFT I
Exercise Sheet 2

Lecturer: Prof. Gehrmann
Assistants: Pier Francesco Monni, Erich Weihs,
Stephan Bühler, Mathias Ritzmann, Gabriel Abelof
www-theorie.physik.uzh.ch/~pfmonni/QFTI_HS10/

Issued : 27.09.2010
Due : 04.10.2010
Discussion : 06.10.2010

Exercise 1 *Time invariance operator for Dirac spinors*
Show, that by demanding invariance under time reversal of the Dirac equation coupled to an electromagnetic field one obtains a time-reversal operator $S(T)$ with

$$\psi'(t' = -t) = S(T)\psi^*(t) \quad \text{and} \quad S(T) = i\gamma^1\gamma^3.$$

Use that $\vec{A}'(\vec{x}, t) = -\vec{A}(\vec{x}, -t)$ and $\phi(\vec{x}, t) = \phi(\vec{x}, -t)$.

Exercise 2 *Explicit form of Dirac spinors*
With the help of Pauli-spinors $\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ the spinors $u_\pm(p)$ and $v_\pm(p)$ can be written as

$$u_\pm(p) = \sqrt{p^0 + m} \begin{pmatrix} \chi_\pm \\ \frac{\vec{\sigma}\cdot\vec{p}}{p^0+m}\chi_\pm \end{pmatrix} \tag{1}$$

$$v_\pm(p) = \sqrt{p^0 + m} \begin{pmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{p^0+m}\chi_\mp \\ \chi_\mp \end{pmatrix}. \tag{2}$$

Derive (1) and (2) from the relations

$$\begin{aligned} u_\pm(p) &= A(p)\,[\not{p} + m]\; u_\pm(m, \vec{0}), \\ v_\pm(p) &= A(p)\,[-\not{p} + m]\; v_\pm(m, \vec{0}), \\ A(p) &= 1/\sqrt{2m(p^0 + m)}. \end{aligned}$$

In addition, show that one can obtain equations (1) and (2) by applying an appropriate Lorentz boost to the solutions for the Dirac equation for a particle at rest.

Exercise 3 *Orthogonality relations for Dirac spinors*
Show, that the spinors $u(p)$ and $v(p)$ fulfill the following orthogonality relations:

$$\bar{u}_r(p)u_s(p) = 2m\,\delta_{rs} \quad , \quad \bar{v}_r(p)v_s(p) = -2m\,\delta_{rs}$$
$$\bar{u}_r(p)v_s(p) = 0 \quad , \quad \bar{v}_r(p)u_s(p) = 0$$