QFT I
Exercise Sheet 6

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Exercise 1 [*Hamiltonian operator and Poynting Vector*]
We have shown that the Energy-Momentum tensor for the electromagnetic field reads:

$$T^{\mu\nu} = -F^{\mu\rho}F_{\rho}^{\nu} + \frac{1}{4}g^{\mu\nu}F^{\rho\sigma}F_{\rho\sigma}. \tag{1}$$

Using the Fourier reppresentation of the photon field A^μ

$$A_\mu = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\frac{1}{\sqrt{2p_0}}\sum_\lambda \left(a_p^\lambda \epsilon_\mu^\lambda(p)e^{-ipx} + a_p^{\lambda\dagger}\epsilon_\mu^{\lambda *}(p)e^{ipx}\right) \tag{2}$$

show that the Energy Operator and the Poynting Vector are

$$:E: \;=\; \int \frac{\mathrm{d}^3p}{(2\pi)^3}\, p_0 \sum_{\lambda=1}^{2} a_p^{\lambda\dagger}a_p^\lambda \tag{3}$$

$$:\vec{P}: \;=\; \int \frac{\mathrm{d}^3p}{(2\pi)^3}\, \vec{p} \sum_{\lambda=1}^{2} a_p^{\lambda\dagger}a_p^\lambda. \tag{4}$$

Exercise 2 [*Gauge-fixed electromagnetic Lagrangian*]
Given the following Lagrangian density

$$\mathcal{L}_\lambda = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\lambda}{2}\left(\partial_\nu A^\nu\right)^2, \tag{5}$$

show that the corresponding equation of motion is

$$\partial_\nu\partial^\nu A^\mu - (1-\lambda)\partial^\mu\left(\partial_\nu A^\nu\right) = 0. \tag{6}$$

Exercise 3 [*Planck's radiation Law*]
Consider a system made of atoms and a radiation field wich can freely exchange energy by the reversible process

$$A \rightleftharpoons \gamma + B \tag{7}$$

in such a way that thermal equilibrium is established. Defining the populations of the upper and lower atomic levels as $N(A)$ and $N(B)$ respectively, we have the following equilibrium condition

$$N(B)\, w_{abs} = N(A)\, w_{emis} \tag{8}$$

$$\frac{N(B)}{N(A)} = \frac{e^{\frac{-E_B}{kT}}}{e^{\frac{-E_A}{kT}}}, \tag{9}$$

where w_{abs} and w_{emis} are the transition probabilities for $B + \gamma \to A$ and $A \to B + \gamma$ respectively. Compute the ratio w_{emis}/w_{abs} in terms of the number of photons $n(\vec{k}, \alpha)$ and, using the previous equations, find an explicit expression for $n(\vec{k}, \alpha)$ itself. What you get is the Planck's radiation law.

Exercise 4 [*Photon propagator*]
Show that the following identity for the photon field operator and its propagator holds (use the Feynman gauge):

$$\left[A_\mu(x), A_\nu(x')\right] = i(-g_{\mu\nu})\Delta(x - x'). \tag{10}$$