QFT I
Exercise Sheet 7

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Exercise 1 [*Time-evolution operator*]

The time-evolution operator $U(t, t_0)$ creates a connection between the states of time t_0 and t (in the interaction picture):

$$\psi_I(t) = U(t, t_0)\psi_I(t_0) \quad , \quad U(t_0, t_0) = \mathbf{1} \; .$$

It solves the differential equation

$$i\partial_t \, U(t, t_0) = H_I(t) \, U(t, t_0) \; . \tag{1}$$

- Show that $U(t, t_0)$ is unitary.
- Show that $U(t, t_0)$ is the transformation operator between the Heisenberg and the interaction picture, i.e.:

$$\begin{aligned} O_I(t) &= U(t, 0) O_H(t) U^{-1}(t, 0), \\ |\alpha, t\rangle_I &= U(t, 0) \, |\alpha\rangle_H \, . \end{aligned}$$

- Let now $t > t_0$. Show that the solution of equation (1) is given by the so-called "Neumann series":

$$\begin{aligned} U(t, t_0) &= \mathbf{1} + (-i) \int_{t_0}^{t} dt_1 H_I(t_1) \\ &+ (-i)^2 \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \, H_I(t_1) H_I(t_2) \\ &+ \ldots \\ &+ (-i)^n \int_{t_0}^{t} dt_1 \ldots \int_{t_0}^{t_{n-1}} dt_n \, H_I(t_1) H_I(t_2) \ldots H_I(t_n) \\ &+ \ldots \end{aligned} \tag{2}$$

- Show that (2) can be written as

$$U(t,t_0) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{t_0}^{t} dt_1 \ldots \int_{t_0}^{t} dt_n \, T\left(H_I(t_1) \ldots H_I(t_n)\right) \ , \tag{3}$$

where T is the time-ordering operator.

Exercise 2 [*Wick's theorem and Feynman rules for $\lambda\phi^4$-theory*]

- Calculate the second-order contribution in λ to $\langle\Omega|T(\phi_H(x)\phi_H(y))|\Omega\rangle$ in $\lambda\phi^4$-theory, i.e. (before normalization):

$$\langle 0|T\left(\phi(x)\phi(y)\,\frac{1}{2!}(-i)^2 \int dt_1 dt_2 H_I(t_1) H_I(t_2)\right)|0\rangle$$

  (i) directly by using Wick's theorem,

  (ii) employing the Feynman rules in real space.

- Transform the propagators in this expression to momentum space and evaluate momentum conservation.