QFT I
Exercise Sheet 8

Lecturer: Prof. Thomas Gehrmann
Assistants: Pier Francesco Monni, Erich Weihs, Stephan Bühler, Mathias Ritzmann, Gabriel Abelof
www-theorie.physik.uzh.ch/~pfmonni/QFTI_HS10/

Issued : 08.11.2010
Due : 15.11.2010
Discussion : 18.11.2010

Exercise 1 [*Optical theorem*]

Deduce from the unitarity of the S-matrix, $SS^\dagger = 1$, that:

$$T_{fi} - T_{if}^* = i(2\pi)^4 \sum_n \delta(p_f - p_n) T_{fn} T_{in}^*$$

Exercise 2 [*Møller scattering*]

Calculate the $\mathcal{O}(e^2)$ contribution to the scattering matrix element S_{fi} for Møller scattering:

$$|i\rangle = |e^-(p_1, s_1) e^-(p_2, s_2)\rangle \,, \qquad \langle f| = \langle e^-(p_2', s_2') e^-(p_1', s_1')|$$

(i) through direct evaluation in x-space.

(ii) using the momentum-space Feynman rules for QED.

Exercise 3 [*Bhabha scattering*]

Calculate the $\mathcal{O}(e^2)$ contribution to the scattering matrix element S_{fi} for Bhabha scattering,

$$|i\rangle = |e^+(p_1, s_1) e^-(p_2, s_2)\rangle \,, \qquad \langle f| = \langle e^-(p_2', s_2') e^+(p_1', s_1')| \,,$$

using the momentum space Feynman rules for QED.

Exercise 4 [*Feynman rules DIY*]

In this exercise, you will derive some of the Feynman rules for *scalar QED*, a theory which consists of a complex scalar field ϕ and a minimally coupled vector field A_μ. The Lagrangian reads

$$\mathcal{L} = (D_\mu\phi)(D^\mu\phi)^\dagger - m^2\phi^\dagger\phi - \frac{1}{4}\lambda(\phi^\dagger\phi)^2 - \frac{1}{4}F^{\mu\nu}F_{\mu\nu},$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$, and $D_\mu = \partial_\mu + ieA_\mu(x)$ is the covariant derivative coupling ϕ and A_μ.

What kind of propagators and interaction vertices does this theory have? What are the expressions for the propagators of $A_\mu(x)$ and $\phi(x)$? The Feynman rules for the four particle interaction vertices are shown in figure 1. Can you argue where these expressions come from?

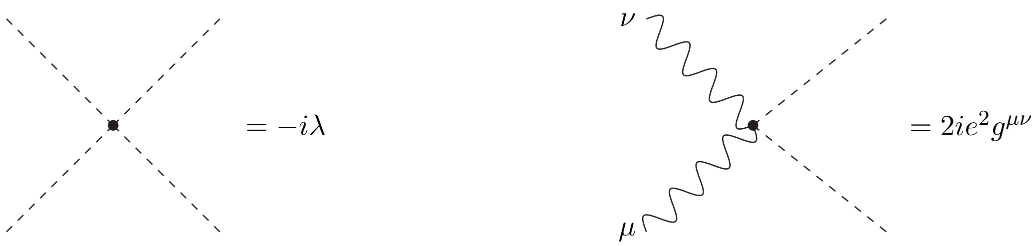

Figure 1: Feynman rules for the four particle vertices.

Note: There is also a three particle vertex, arising from a term $\sim ie(\partial_\mu\phi)A^\mu\phi^\dagger$. In figure 2 we stated the Feynman rules for completeness. However in this course you will not learn how to derive Feynman rules for terms with partial derivatives, this will be treated in QFT II.

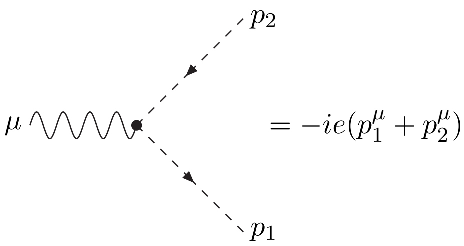

Figure 2: Feynman rules for the three particle vertex.