QFT I
Exercise Sheet 9

ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Lecturer: Prof. Thomas Gehrmann
Assistants: Pier Francesco Monni, Erich Weihs,
Stephan Bühler, Mathias Ritzmann, Gabriel Abelof
www-theorie.physik.uzh.ch/~pfmonni/QFTI_HS10/

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Exercise 1 [$2 \to 2$ *scattering*]

Consider a $2 \to 2$ particle scattering process $(1 + 2 \to 3 + 4)$. Calculate the following in the center-of-mass frame:

- The energy E_i^* of the particles and their momenta $|\mathbf{p}|$ before and $|\mathbf{p}'|$ after the collision. Determine the asymptotic behaviour of these for $s \gg m_i^2$.
- Show for the scattering angle Θ^*:

$$\cos\Theta^* = \frac{s(t-u) + (m_1^2 - m_2^2)(m_3^2 - m_4^2)}{\sqrt{\lambda(s, m_1^2, m_2^2)}\sqrt{\lambda(s, m_3^2, m_4^2)}}$$

  with

$$\lambda(s, m_1^2, m_2^2) = (s - m_1^2 - m_2^2)^2 - 4m_1^2 m_2^2 = s^2 + m_1^4 + m_2^4 - 2sm_1^2 - 2sm_2^2 - 2m_1^2 m_2^2.$$

- Show $s + t + u = \sum_i m_i^2$.
- Determine $t_{\min}$ and $t_{\max}$ from the condition $|\cos\Theta^*| \leq 1$. Determine the asymptotic behaviour of $t_{\min}$ and $t_{\max}$ for $s \gg m_i^2$.

Exercise 2 [$e^+e^- \to \mu^+\mu^-$]

In this exersise, you are going to calculate the cross section for $e^+ + e^- \to \mu^+ + \mu^-$. Proceed as follows:

(i) Draw all Feynman diagrams which contribute to this process at the given order (in our example there is only one diagram, see below).

(ii) Read off the amplitude for every graph using the Feynman rules.

(iii) Take the norm squared of the sum of all amplitudes.

(iv) Sum over the spins/polarisations of the outgoing particles and average over the spins/polarisations of the incoming particles to transform the terms in the norm squared of the sum of the amplitudes into a sum over traces (in our example there is only one trace).

(v) Simplify the expression using trace identities.

(vi) Multiply by the flux factor.

(vii) Express the result in a frame of your choice (for example in the center-of-mass frame).